

Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 1 (a) Sketch the graph of $y = |2x + 1|$.

[1]

- (b) Solve the inequality $3x + 5 < |2x + 1|$.

[3]

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- 2 On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z| \leq 3$, $\operatorname{Re} z \geq -2$ and $\frac{1}{4}\pi \leq \arg z \leq \pi$. [4]

- 3** Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4** Solve the equation $\tan(x + 45^\circ) = 2 \cot x$ for $0^\circ < x < 180^\circ$. [5]

[illegible]

- 5** The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

- (a) Find $\frac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ . [3]

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- (b) State the least positive integer n such that both $\operatorname{Im} w^n = 0$ and $\operatorname{Re} w^n > 0$. [1]

6 (a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4 \theta$.

[4]

[illegible]

[Turn over

- 7** The equation of a curve is $y = \frac{x}{\cos^2 x}$, for $0 \leq x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1} \left(\sqrt[3]{\frac{\cos a + 2a \sin a}{12}} \right)$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Verify by calculation that a lies between 0.9 and 1.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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- 8 In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t , the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

- (a) Solve the differential equation, obtaining a relation between x , t and k . [5]

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- (b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

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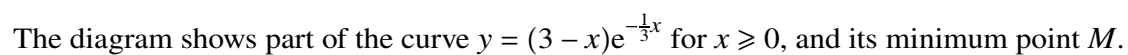
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- [5]

[illegible]

- (b) Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e . [5]

[illegible]

10 Let $f(x) = \frac{2x^2 + 7x + 8}{(1+x)(2+x)^2}$.

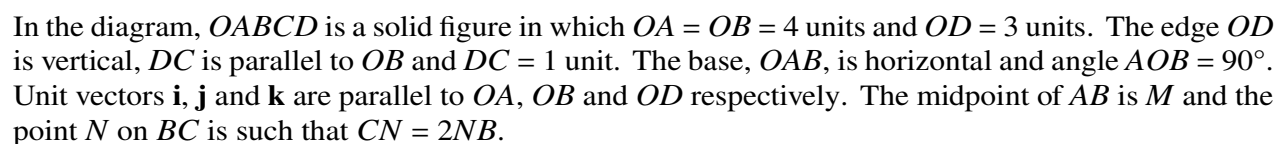
(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]



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- This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

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- (c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

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